B.A./B.Sc. 5th Semester (Honours) Examination, 2022 (CBCS)

Subject : Mathematics

Course : BMH5DSE11

(Linear Programming)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions from the following: 2×10=20

(a) Prove that a hyperplane and a closed half space in E^n are unbounded closed convex sets.

(b) Is the set $X = \{(x, y) \in E^2 | x + y = 2\}$ convex? Justify.

(c) Write down linear programming problem (L.P.P.) associated with a transportation problem with 'm' origins and 'n' destinations.

(d) Write down the advantage of Hungarian method over simplex method for solving an Assignment Problem.

(e) What do you mean by basic solution of an L.P.P.?

(f) Write down the algorithm to solve a balanced transportation problem by North-West corner method.

(g) Distinguish between extreme point and boundary point for a given set with a suitable example.

(h) What do you mean by degeneracy in transportation problem?

(i) Find the convex hull of the following set:

Half lines: $x_2 = 0, x_1 \geq 0$ and $x_2 \geq 0, x_1 = 0$ in E^2

(j) Solve the game whose payoff matrix is given by

Player B

Player A		B_1	B_2	B_3
	A_1	1	3	3
	A_2	0	–4	–3
	A_3	1	5	–1

(k) Explain the Maximin and Minimax principle used in Game Theory.

(l) Define saddle point. Is it necessary that a problem on Game Theory should always possess a saddle point?

(m) State dominance properties of a zero sum two person rectangular game.

(n) If $x^T = (x_1, x_2, .., x_n)$ is a feasible solution of the primal problem and $v^T = (v_1, v_2, .., v_n)$ is a feasible solution of the dual problem, then $cx = bv$.

(o) Prove that the solution of a transportation problem is never unbounded.

2. Answer *any four* questions from the following: 5×4=20

(a) Show that a feasible solution to a Transportation Problem is basic if the corresponding cells in the transportation table do not contain a loop.

(b) If $x_1 = 1, x_2 = 1, x_3 = 2$ is a feasible solution of the equations

$$x_1 + 2x_2 + 3x_3 = 9$$
$$2x_1 - x_2 + x_3 = 3,\ x_1,\ x_2,\ x_3 \geq 0$$

then reduce the feasible solution to a basic feasible solution of the above equations.

(c) Solve the following problem by two phase method:

Maximize $Z = 5x_1 + 3x_2$

subject to $3x_1 + x_2 \leq 1$

$3x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$

(d) Solve the following L.P.P.

Maximize $z = 5x_1 + 4x_2$

subject to $3x_1 + 4x_2 \leq 24$

$3x_1 + 2x_2 \leq 18$

$x_2 \leq 5,\ x_1, x_2 \geq 0$

(e) Solve the game problem with the following pay-off matrix reducing it into 2 × 2 problem using the dominance property:

0	0	0	0	0
4	2	0	2	1
4	3	1	3	2
4	3	4	−1	2

(f) (i) Show that if any of the constraints in the primal problem be a perfect equality, the corresponding dual variable is unrestricted in sign.

(ii) What is an unbiased assignment problem? How do you solve it? 3+(1+1)

3. Answer *any two* questions from the following: 10×2=20

(a) (i) Use the dual simplex method to solve the L.P.P.:

Maximize $Z = x_1 + x_2$

subject to $2x_1 + x_2 \geq 2$

$-x_1 - x_2 \geq 1$

$x_1,\ x_2 \geq 0$.

(ii) If a transportation problem has m origins and n destinations, then prove that the number of basic variables in that transportation problem is at most $(m + n - 1)$. Also show that the transportation problem always has a feasible solution. 5+(3+2)

(b) (i) Consider the problem of assigning four operators to four machines. The assignment costs in rupees are given here. Operator 1 cannot be assigned to machine III and operator 3 cannot be assigned to machine IV. Find the optimal cost of assignment.

	I	II	III	IV
1	5	5	–	2
2	7	4	2	3
3	9	3	5	–
4	7	2	6	7

(ii) A company has factories *A*, *B* and *C* which supply to the warehouses at *D*, *E*, *F* and *G*. Monthly factory capacities are 160, 150 and 190 units respectively. Monthly warehouse requirements are 80, 90, 110 and 160 units, respectively. Unit shopping cost (in ₹) are as follow:

		To			
		D	*E*	*F*	*G*
From	*A*	42	48	38	37
	B	40	49	52	51
	C	39	38	40	43

Determine the optimum distribution for his company to minimize the shopping cost. 5+5

(c) (i) Solve the following 2×4 game graphically:

		Player B			
Player A	1	3	–3	7	
	2	5	4	–6	

(ii) Prove that every extreme point of the convex set of all feasible solutions of the system $Ax = b,\ x \geq 0$ corresponds to a basic feasible solution.

(iii) Find the extreme points, if any, of the set $X = \{(x, y)/x^2 + y^2 \leq 16\}$. 5+3+2

(d) (i) Solve the following Travelling Salesman Problem:

		To			
		A	B	C	D
From	A	∞	46	16	40
	B	41	∞	50	40
	C	82	32	∞	60
	D	40	40	36	∞

(ii) Using simplex method, solve the following system of linear questions: 5+5

$$x_1 + x_2 = 6$$
$$2x_1 + x_2 = 4$$

———

B.A./B.Sc. 5th Semester (Honours) Examination, 2022 (CBCS)

Subject : Mathematics

Course : BMH5DSE12

(Number Theory)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

2×10=20

1. Answer *any ten* questions:

   (a) Show that the congruence $x^2 \equiv 1 \pmod 8$ has exactly four solutions.

   (b) Is the Diophantine equation $6x + 3y = 22$ solvable? If not, state the reason.

   (c) If $n > 2$, then prove that $\phi(n)$ is even.

   (d) If a be a primitive root of m and $(x, m), (y, m) = 1$, then prove that $\mathrm{ind}_a^1 = 0$ and $\mathrm{ind}_a^a = 1$.

   (e) Solve, if possible, $x^8 \equiv 10 \pmod{11}$.

   (f) Show that the converse of Fermat's theorem is not true.

   (g) Show that $3.5^{2n+1} + 2^{3n+1} \equiv 0 \pmod{17}\ \forall\, n \geq 1$.

   (h) Find the least +ve residue of $11^{30} \pmod 4$.

   (i) Calculate $\phi(1024)$.

   (j) Find the remainder when 2 (26 !) is divided by 29.

   (k) Solve $x^2 \equiv 14 \pmod{5^3}$.

   (l) If n be a perfect number, then show that $\sum_{d/n} \frac{1}{d} = 2$.

   (m) Show that for any integer a, $a^3 \equiv 0, 1$ or $6 \bmod (7)$.

   (n) Prove that $\phi(2^n - 1)$ is a multiple of n for any prime $n > 1$.

   (o) Define primitive root with an example.

5×4=20

2. Answer *any four* questions:

   (a) Find three primitive roots of 26. 5

   (b) Solve for integer solution: $8x - 27y = 125$ 5

   (c) Prove that if a is prime to b then $a^2 + b^2$ is prime to a^2b^2. 5

(d) Solve the congruences $x \equiv 5 \pmod 4$, $x \equiv 3 \pmod 7$ and $x \equiv 2 \pmod 9$. 5

(e) Show that $\lfloor 28 + 233 \equiv 0 \pmod{899}$. 5

(f) Prove that Möbius function $\mu(n)$ is multiplicative. 5

3. Answer *any two* questions: 10×2=20

(a) (i) Solve $3x^4 \equiv 5 \pmod{11}$.

(ii) State and prove Chinese Remainder theorem. 5+5

(b) (i) Find the values of the Legendre symbols: $\left(\frac{199}{3}\right), \left(\frac{5}{199}\right)$

(ii) Show that $x^2 \equiv 219 \pmod{419}$ is not solvable. 5+5

(c) Prove that any primitive solution of $x^2 + y^2 = z^2$ is of the form $(2ab, a^2 - b^2, a^2 + b^2)$ for some integers a and b such that $a > b > 0$ satisfying $(a, b) = 1$ and $a + b \equiv 1 \pmod 2$. 10

(d) (i) If p is a prime number, then prove that $\lfloor p-1 \equiv p-1 \pmod{1 + 2 + 3 + \ldots + p - 1}$.

(ii) The sum of two positive integers is 100. If one is divided by 7 has remainder 1, and if the other is divided by 9 has the remainder 7, then find the numbers. 5+5

———

B.A./B.Sc. 5th Semester (Honours) Examination, 2022 (CBCS)

Subject : Mathematics

Course : BMH5DSE13

(Point Set Topology)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* questions: 2×10=20

(a) Let X be an infinite set with the co-finite topology. Show that X is connected.

(b) Give an example to show that the union of two topologies on a set need not be a topology.

(c) Prove that the usual topology of $\mathbb{R}$ is strictly weaker than the lower limit topology on $\mathbb{R}$.

(d) Prove that every closed subset of a compact space is compact.

(e) Examine whether the set $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1, x > 0, y > 0\}$ is compact in $\mathbb{R}^2$.

(f) Let A be a compact subset of a metric space (X, d). Show that the derived set A' of A is compact.

(g) Using the definition of compactness, show that the open interval $(0, 1)$ is not compact in the real number space $\mathbb{R}$ with usual metric.

(h) Define cardinality of a set.

(i) State Zorn's lemma.

(j) Define a locally connected space. Give an example of a locally connected space which is not connected. 1+1

(k) Prove that the real line $\mathbb{R}$ and the open interval $(0, 1)$ in $\mathbb{R}$ have the same cardinality.

(l) If $A \subset X, X$ a topological space, show that $\overline{(X - A)} = X - \text{int}\, A$.

(m) Examine if the set $\left\{1, 2, \frac{1}{2}, 3, \frac{1}{3}, \ldots, 2022, \frac{1}{2022}\right\}$ is connected in the space of reals with usual topology.

(n) Define a homeomorphism between two topological spaces. When is a topological property said to be topological invariant? 1+1

(o) Let $f: X \to \mathbb{R}$ be a non-constant continuous function, where X is connected. Prove that $f(X)$ is uncountable.

2. Answer *any four* questions: 5×4=20

(a) State and prove Schröder–Bernstein theorem. 1+4

(b) If u is an infinite cardinal number, prove that $u + u = u$. 5

(c) If α, β and γ are order types with $\alpha < \beta$ and $\beta < \gamma$, then prove that $\alpha < \gamma$. 5

(d) Let A be a subset of a topological space (X, τ) and $x_0 \in X$. If there exists a sequence $\{x_n\}$ in A such that $\{x_n\}$ converges to x_0, then prove that $x_0 \in \overline{A}$, where $\overline{A}$ is the closure of A. Give an example to show that the converse need not hold. 2+3

(e) Define a Lebesgue number. Prove that in a sequentially compact metric space, every open cover has a Lebesgue number. 1+4

(f) Prove that a subset of $\mathbb{R}$ is connected if and only if it is either a singleton or an interval. 2+3

3. Answer *any two* questions: 10×2=20

(a) (i) Let X and Y be two topological spaces and $f: X \to Y$ be a function. Prove that f is continuous on X if and only if $f(\overline{A}) \subset \overline{f(A)}$ for every $A \subset X$.

(ii) Prove that the image of a locally connected space (X, τ) under a mapping which is both open and continuous is locally connected. Give an example to show that the continuous image of a locally connected space need not be locally connected. (2+3)+(4+1)

(b) (i) Prove that a topological space (X, τ) is compact if and only if every family of closed subsets of X having FIP has nonempty intersection.

(ii) If A is a compact set in a metric space (X, d) with diameter $\delta(A)$, prove that $\exists$ a pair of points x and y of A such that $d(x, y) = \delta(A)$. Is it true if A be non-compact? Justify your answer. (2½+2½)+(4+1)

(c) (i) Let A and B be two subsets of a topological space (X, τ). Prove that $\text{int}\,(A \cap B) = \text{int}\,A \cap \text{int}\,B$. Give an example to show that $\text{int}\,(A \cup B) \neq \text{int}\,A \cup \text{int}\,B$.

(ii) If D is dense in a topological space (X, τ), then prove that $\overline{V \cap D} = \overline{V}, \forall\, V \in \tau$. Does the converse hold? Support your answer.

(iii) Prove that a totally bounded metric space is bounded. (3+1)+(2+1)+3

(d) (i) Let u be the cardinal number of the set U and $P(U)$ be the power set of U. Prove that

1. $\overline{\overline{P(U)}} = 2^u$ and
2. $u < 2^u$.

(ii) Prove that $2^a = c$, where a and c denote the cardinality of the sets $\mathbb{N}$ and $\mathbb{R}$ respectively. (3+3)+4

———